7.1 Oblique Triangles and the Law of Sines

Congruency and Oblique Triangles ▪ Derivation of the Law of Sines ▪ Solving SAA and ASA Triangles (Case 1) ▪ Area of a Triangle

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Objective #1: Students will be able to use the Law of Sines to solve oblique triangle problems.

SC #1: I can solve LOS problems when given two angles and a side in an oblique triangle.

SC#2: I can find area of an oblique triangle using a formula involving Sine.
Congruence Axioms

**Side-Angle-Side (SAS)**

If two sides and the included angle of one triangle are equal, respectively, to two sides and the included angle of a second triangle, then the triangles are congruent.

**Angle-Side-Angle (ASA)**

If two angles and the included side of one triangle are equal, respectively, to two angles and the included side of a second triangle, then the triangles are congruent.

**Side-Side-Side (SSS)**

If three sides of one triangle are equal, respectively, to three sides of a second triangle, then the triangles are congruent.
Oblique Triangles

- **Oblique triangle** - A triangle that is not a right triangle

- The measures of the three sides and the three angles of a triangle can be found if at least one side and any other two measures are known.
Data Required for Solving Oblique Triangles

**Case 1** One side and two angles are known (SAA or ASA).

**Note**

*If three angles of a triangle are known, unique side lengths cannot be found because AAA assures only similarity, not congruence.*

between the two sides are known (SAS).

**Case 4** Three sides are known (SSS).
Start with an oblique triangle, either acute or obtuse.

Let \( h \) be the length of the perpendicular from vertex \( B \) to side \( AC \) (or its extension).

Then \( c \) is the hypotenuse of right triangle \( ABD \), and \( a \) is the hypotenuse of right triangle \( BDC \).
Derivation of the Law of Sines

In triangle $ADB$, 
\[ \sin A = \frac{h}{c} \text{ or } h = c \sin A \]

In triangle $BDC$, 
\[ \sin C = \frac{h}{a} \text{ or } h = a \sin C \]

Since $h = c \sin A$ and $h = a \sin C$, 
\[ a \sin C = c \sin A \]
\[ \frac{a}{\sin A} = \frac{c}{\sin C} \]

Similarly, it can be shown that 
\[ \frac{a}{\sin A} = \frac{b}{\sin B} \text{ and } \frac{b}{\sin B} = \frac{c}{\sin C}. \]
Law of Sines

In any triangle $ABC$, with sides $a$, $b$, and $c$,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
Example 1

**USING THE LAW OF SINES TO SOLVE A TRIANGLE (SAA)**

Solve triangle $ABC$ if $A = 32.0^\circ$, $B = 81.8^\circ$, and $a = 42.9$ cm.

\[
\frac{a}{\sin A} = \frac{b}{\sin B} \quad \text{Law of sines}
\]

\[
\frac{42.9}{\sin 32.0^\circ} = \frac{b}{\sin 81.8^\circ}
\]

\[
b = \frac{42.9 \sin 81.8^\circ}{\sin 32.0^\circ} \approx 80.1 \text{ cm}
\]

\[
A + B + C = 180^\circ
\]

\[
C = 180^\circ - A - B
\]

\[
C = 180^\circ - 32.0^\circ - 81.8^\circ = 66.2^\circ
\]

Use the Law of Sines to find $c$.

\[
\frac{a}{\sin A} = \frac{c}{\sin C}
\]

\[
\frac{42.9}{\sin 32.0^\circ} = \frac{c}{\sin 66.2^\circ}
\]

\[
c = \frac{42.9 \sin 66.2^\circ}{\sin 32.0^\circ} \approx 74.1 \text{ cm}
\]
Jerry wishes to measure the distance across the Big Muddy River. He determines that $C = 112.90°$, $A = 31.10°$, and $b = 347.6$ ft. Find the distance $a$ across the river.

First, find the measure of angle $B$.

$B = 180° - A - C = 180° - 31.10° - 112.90° = 36.00°$

Now use the Law of Sines to find the length of side $a$.

\[
\frac{a}{\sin A} = \frac{b}{\sin B} \quad \frac{a}{\sin 31.10°} = \frac{347.6}{\sin 36.00°} \quad a = \frac{347.6 \sin 31.10°}{\sin 36.00°} \approx 305.5 \text{ ft}
\]

The distance across the river is about 305.5 feet.
Two ranger stations are on an east-west line 110 mi apart. A forest fire is located on a bearing N 42° E from the western station at A and a bearing of N 15° E from the eastern station at B. How far is the fire from the western station?

First, find the measures of the angles in the triangle.

\[
m\angle BAC = 90° - 42° = 48°
\]
\[
m\angle ABC = 90° + 15° = 105°
\]
\[
m\angle C = 180° - 105° - 48° = 27°
\]
Now use the Law of Sines to find $b$.

\[
\frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
\frac{b}{\sin 105^\circ} = \frac{110}{\sin 27^\circ}
\]

\[
b = \frac{110 \sin 105^\circ}{\sin 27^\circ} \approx 234 \text{ mi}
\]

The fire is about 234 miles from the western station.
Law of sines

Classwork

- do two problems on handout
In any triangle $\triangle ABC$, the area $A$ is given by the following formulas:

$A = \frac{1}{2} bc \sin A \quad A = \frac{1}{2} ac \sin B \quad A = \frac{1}{2} ab \sin C$

**Note**

If the included angle measures $90^\circ$, its sine is 1, and the formula becomes the familiar

$A = \frac{1}{2} bh$. 
Find the area of triangle ABC.

\[ A = \frac{1}{2}ac \sin B \]
\[ = \frac{1}{2}(34.0)(42.0)\sin 55^\circ 10' \]
\[ \approx 586 \text{ ft}^2 \]

Find the area of the triangle, ABC with \( A = 72^\circ, b = 16 \) and \( c = 10 \).

\[ A = \frac{1}{2}bc \sin A \]
\[ A = \frac{1}{2}(16)(10)\sin 72^\circ \]
\[ A \approx 76.1 \text{ ft}^2 \]
Find the area of triangle $ABC$ if $A = 24°40'$, $b = 27.3$ cm, and $C = 52°40'$.

Before the area formula can be used, we must find either $a$ or $c$.

Draw a diagram.

$B = 180° – 24°40′ – 52°40′ = 102°40′$

$$\frac{a}{\sin A} = \frac{b}{\sin B} \quad \frac{a}{\sin 24°40'} = \frac{27.3}{\sin 102°40'}$$

$a = \frac{27.3 \sin 24°40'}{\sin 102°40'}$

Now find the area.

$$A = \frac{1}{2} ab \sin C = \frac{1}{2} (11.7)(27.3) \approx 127 \text{ cm}^2$$

Caution

Whenever possible, use given values in solving triangles or finding areas rather than values obtained in intermediate steps to avoid possible rounding errors.
Area Classwork

- do problem on handout